P2P Networks – Exercise
Solution For Exercise # 13

Final

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Graph Theory

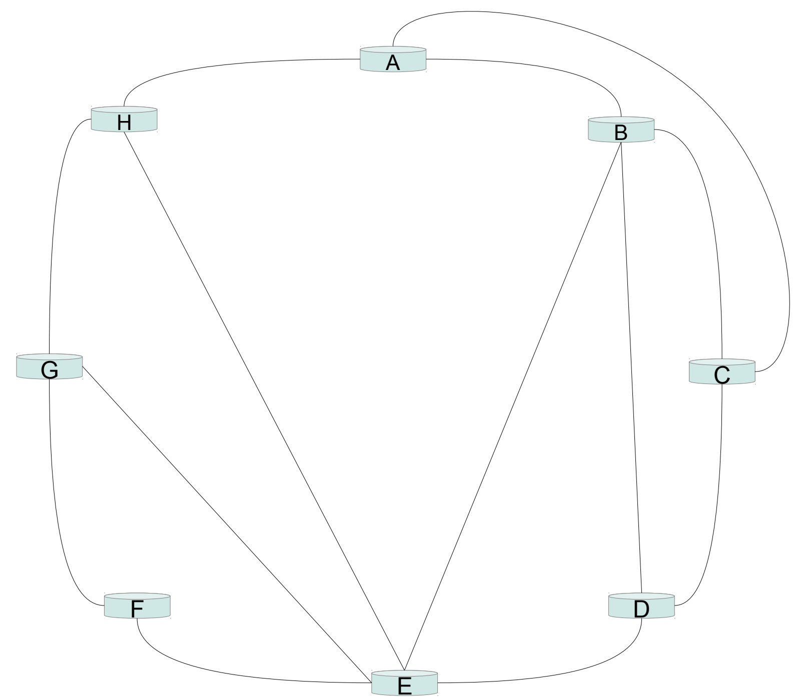
13.1) Network Models

- Graphs/Networks
  - Cluster coefficient (CC)
  - Shortest path length (SPL)
- Random Networks (Erdös-Reyni (RR) Model)
  - Nodes haves have no preference for interconnecting with each others
  - Small CC and small SPL
- Small world Networks
  - Nodes connect with each other based on some preferences
  - Large CC and small SPL
  - Examples:including road maps, food chains, electric power grids
- Scale-free Networks
  - Small CC decreases with and SPL
  - Ultra-small worlds. The shortest paths become significantly smaller and scale as L~LogLog N Cohen and Havlin[1][2]
  - Examples: World Wide Web links, biological networks, and social networks

13.1) All Pairs Shortest Path (APSP)

- Input: a graph, G
- Output: For each pair of nodes u, v ∈ V, the distance from u to v and a shortest path

	A	B	C	D	E	F	G	H
A	0	1	1	2	2	3	2	1
B	1	0	1	1	1	2	2	2
C	1	1	0	1	2	3	3	2
D	2	1	1	0	1	2	2	2
E	2	1	1	1	0	1	1	1
F	3	2	3	2	1	0	1	2
G	2	2	3	2	1	1	0	1
H	1	2	2	2	1	2	1	0



13.1) Basic Concepts

- Rich-club connectivity
  - The metric which shows that small number of nodes with higher node degree, are connected with each other e.g., Internet topology at the Autonomous System (AS) levelThe

- Assortativity
  - Preference of nodes to connect to other nodes that are similar (nodes with same degree) e.g., social nets, WSNs

13. 1) Hop Plot

- Given: a graph, G
- Output: for each node v ∈ V find $N_h(v)$ in the neighborhood of h hops, sum the results to find the total neighborhood size N_h for h hops ($N_h = \sum_u h(u)$). The hop-plot is just the plot of N_h versus h
- For h=1, $N_h = 26$
- For h=2, $N_h = 24$
- For h=3, $N_h = 6$

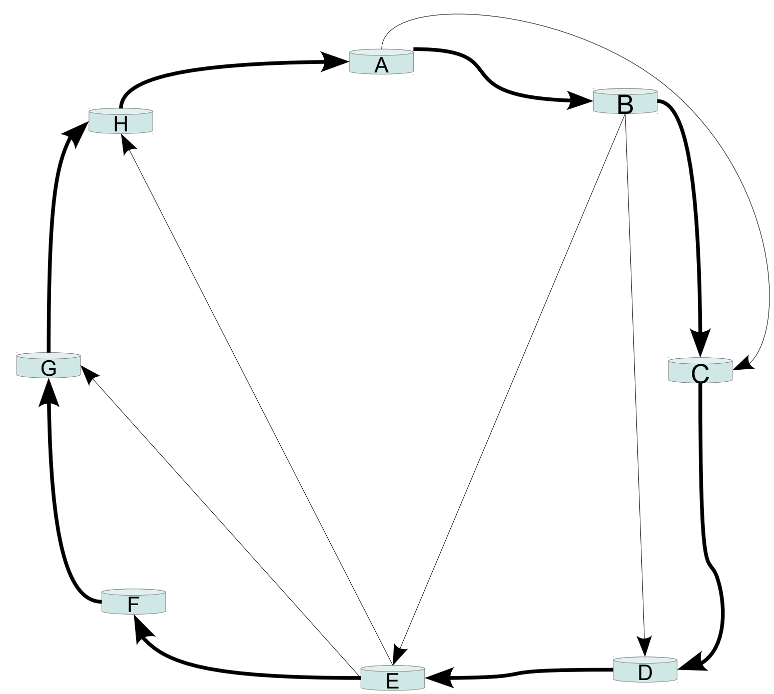
Average/characteristic path length

- Given: a graph, G
- Output: the characteristic path length L(G) of G to be the average of L(v) across all vertices v in V(G).
- $L(G) = (1 * 26 + 2 * 24 + 3 * 6) / 56 = 1.642$

13. 1) Characteristic Routing Length

- Input: weighted graph (simple graph, in the case below) with v, v' $\in$ G
- Output: The minimum number of edges that must be traversed to travel from a vertex v to another vertex v' of a graph G is called the shortest path length (or distance) between v and v'
- CRL = (13+2*14+3*15+4*11+5+3)/56 = 2.59

	A	B	C	D	E	F	G	H
A	0	1	1	2	2	3	3	3
B	3	0	1	1	1	2	2	2
C	4	5	0	1	2	3	3	3
D	3	4	4	0	1	2	2	2
E	2	3	3	4	0	1	1	1
F	3	4	4	5	4	0	2	1
G	1	2	2	3	3	4	0	4
H	2	3	3	4	4	5	1	0



13.2) Connectivity

- Effects of connectivity in power-law graphs
  - Random attacks do not have significant effects
  - Targeted attacks lead to considerable damage i.e., network partitioning attacks

- Connectivity enhances network resilience
  - Attacker need more resources to achieve attack efficiency

13.3) Sub-problems with Graph Theory

- Network motifs
  - Building block of complex graphs/networks
  - Represent inter-connectivity structure

- Load-balancing and fault-tolerance
  - Optimize local structure lead to better load-balancing and fault-tolerance

P2P Networks Exam

- Date: 21.02.2012
- Time: 14:25-16:05
- Place: S2|02/C205

References

- [1] R. Cohen, S. Havlin, and D. ben-Avraham (2002). "Structural properties of scale free networks". Handbook of graphs and networks (Wiley-VCH, 2002) (Chap. 4).
- [2] R. Cohen, S. Havlin (2003). "Scale-free networks are ultrasmall". Phys. Rev. Lett. 90: 058701. Bibcode 2003PhRvL..90e8701C. doi:10.1103/PhysRevLett.90.058701. PMID 12633404.